Design Calculations for the Pressure Relief System and Vent Line

Hermann Nann, Indiana University

- Heat Flux per Unit Area
- Mass Flow Rates
- Fluid Flow and Pressure Drop
- Resistance Coefficient K and Equivalent Length L/D
- Resistance coefficients for target/vacuum relief system
- Resistance coefficients for main vent line in shed/ER2
- Total resistance coefficients
- Maximum pressure
- Conclusion

References

1. Boiloff Rates of Cryogenic Targets Subjected to Catastrophic Vacuum Failure, W. M. Schmitt and C. F. Williamson, Bates Internal Report # 90 – 02, 1990.

2. Flow of Fluids Through Valves, Fittings, and Pipe, Crane Technical Paper No. 410, Crane Co., New York, 1991

Heat Flux per Unit Area

(A) Solar constant: $q = 1.34 \times 10^3$ W/m^2

(B) Heat flux into target vessel: $q = 1.30 \times 10^4$ W/m^2

Calculated under the assumption that the target vessel is surrounded by air (or Ar) using the formulae given in Bates Report #90-2.

It includes:

(1) film boiling of the LH_2 at the inside wall of the target vessel.

(2) conduction of heat through the wall of the target vessel.

(3) convective heat transfer from the surrounding gas to the outside wall of the target vessel.

(C) Heat flux into vacuum vessel: $q = 1.0 \times 10^5$ W/m^2

Estimate includes:

(1) heat capacity of vacuum vessel

(2) condensation/direct freezing of air on the outside surface of the vacuum vessel.

(3) film boiling of the LH_2 at the inside wall of the vacuum vessel.

(4) conduction of heat through the wall of the target vessel.

(5) convective heat transfer from the surrounding gas to the outside wall of the target vessel.

Mass Flow Rates

The mass flow rate is given by:

$$w = \frac{q \cdot A}{h_V}$$

where q = heat flow per unit area
A = area
h_V = enthalpy of vaporization

Enthalpy of vaporization of hydrogen:

$$h_V = 4.45 \times 10^5 \text{ J/kg}$$

(A) Target vessel:

Surface area: $A = 0.5 \text{ m}^2$

assume $q = 1.3 \times 10^4 \text{ W/m}^2$

$\Rightarrow$ $w = 0.032$ lb/sec

assume $q = 1.0 \times 10^5 \text{ W/m}^2$

$\Rightarrow$ $w = 0.25$ lb/sec

Since q is not very well know, use

$w = 0.20$ lb/sec

in all further calculations.

(B) Vacuum vessel:

Surface area: $A = 1.0\ m^2$

assume $q = 1.0 \times 10^5\ W/m^2$

$\Rightarrow$ $w = 0.49$ lb/sec

Use

$w = 0.50$ lb/sec

in all further calculations.

Fluid Flow and Pressure Drop

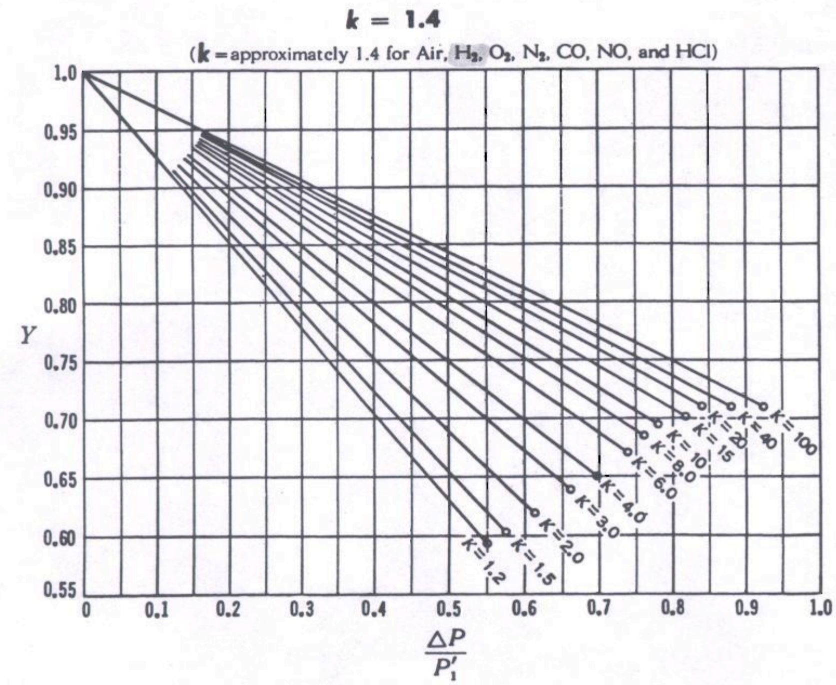
The rate of mass flow through pipes, valves, and fittings is given by the Darcy formula:

$$w = 0.1192 Y d^2 \sqrt{p_1 (p_1 - p_2)\left(\frac{M}{KT}\right)}$$

where
w = mass flow rate [lb/s]
p_1 = inlet (upstream) pressure [psia]
p_2 = outlet (downstream) pressure [psia]
d = inner diameter of pipe [inch]
Y = net expansion factor for compressible flow through orifices, nozzles, or pipe
K = total resistance coefficient for the pipe system
T = absolute temperature of the flowing gas [K]
M = molar mass of the gas [g/mol]

Since the flow in the systems considered here will not be isothermal, the temperature T will be taken to be at the warmest point in the system (room temperature). This will overestimate the pressure p_1, but this will be an error on the side of safety.

Functional dependence of Y versus $(p_1 - p_2)/p_1$:



Limiting Factors For Sonic Velocity
$k = 1.4$

K	$\frac{\Delta P}{P'_1}$	Y
1.2	.552	.588
1.5	.576	.606
2.0	.612	.622
3	.662	.639
4	.697	.649
6	.737	.671
8	.762	.685
10	.784	.695
15	.818	.702
20	.839	.710
40	.883	.710
100	.926	.710

The functional dependence of Y versus $(p_1 - p_2)/p_1$ is linear and can be written in the form

$$Y = 1 - mx$$

where m = absolute value of slope
$x = (p_1 - p_2)/p_1$
$0 \leq x \leq x_{max}$ (The value x_{max} corresponds to sonic flow)

Substituting the linear form for Y into the Darcy equation yields

$$w = 0.1192 d^2 (1 - mx)\left(\frac{p_2}{1 - x}\right)\sqrt{\frac{Mx}{KT}}$$

Squaring both sides of this equation leads to a cubic equation of the form

$$x^3 + ax^2 + bx + c = 0$$

where

$$a = -\frac{\left(w^2 + 2Fm\right)}{Fm^2}$$

$$b = \frac{\left(F + 2w^2\right)}{Fm^2}$$

$$c = -\frac{w^2}{Fm^2}$$

$$F = 0.01423 \cdot \left(\frac{Md^4 p_2^2}{KT}\right)$$

This cubic equation was solved numerically for x.

For subsonic flow, at least one root must lie in the range $0 < x < x_{max}$. If not, then the flow is sonic. The steady-state pressure is then given by

$$p_1 = \frac{p_2}{1 - x}$$

Sonic flow represents the maximum possible flow rate in a piping system. It occurs when the flow velocity equals the velocity of sound in the flowing medium. The mass flow rate at the onset of sonic propagation is given by

$$w_{sonic} = 0.1192 d^2 (1 - m x_{\max}) \left(\frac{p_2}{1 - x_{\max}} \right) \sqrt{\frac{M x_{\max}}{KT}}$$

In order to insure that the flow is always subsonic, w_{sonic} is calculated with the initial value of p_2 which is the atmospheric pressure.

Resistance Coefficient K and Equivalent Length L/D

Pressure losses in a piping system result from a number of system characteristics:

1. Pipe friction, which is a function of the surface roughness of the interior pipe wall, the inside diameter of the pipe, and the fluid velocity, density, and viscosity.
2. Changes in direction of flow path.
3. Obstruction in flow path.
4. Sudden and gradual changes in the cross-section and shape of flow path.

Fluid velocity in a pipe is obtained at the expense of the static head; the decrease in the static head due to the velocity is given by:

$$h_L = \frac{v^2}{2g}$$

This is the definition of the velocity head. Flow through a valve or fitting in a pipe line also causes a reduction in the static head which may be expressed in terms of the velocity head. The resistance coefficient *K* in the equation

$$h_L = K\frac{v^2}{2g}$$

gives the number of velocity heads lost due to a valve or fitting.

The resistance coefficient *K* is always associated with the pipe diameter in which the velocity occurs.

The resistance coefficient *K* can be treated as a constant for any given obstruction (i.e. valve or fitting) in a piping system under all conditions of flow, including laminar flow.

The same loss in straight pipe is expressed by the Darcy equation

$$h_L = \left(f_T \frac{L}{D} \right) \frac{v^2}{2g}$$

From this follows the resistance coefficient *K* for straight pipe as

$$K = f_T \frac{L}{D}$$

where f_T is the friction factor.
The ratio *L/D* is the equivalent length, in pipe diameters of straight pipe that will cause the same pressure drop as the obstruction under the same flow conditions.

The resistance coefficient *K*, for a given line of valves or fittings, varies with size as does the friction factor f_T for straight clean commercial pipe.

Pipe friction data for clean commercial steel pipe with flow in zone of complete turbulence:

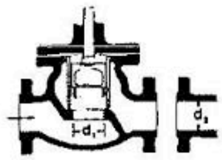
Nominal Size	1.5″	2.0″	2.5″	4.0″	6.0″
Friction Factor f_T	0.021	0.019	0.018	0.017	0.015

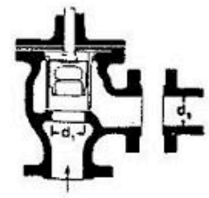
Conversion to different reference diameter:

$$K_a = K_b \left(\frac{d_a}{d_b} \right)^4$$

When a piping system contains more than one size of pipe, this equation allows to express all resistances in terms of one size.

STOP-CHECK VALVES
(Globe and Angle Types)





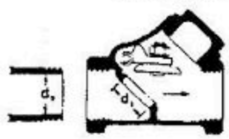
If:
$\beta = 1 \ldots K_1 = 400\, f_T$
$\beta < 1 \ldots K_2 =$ Formula 7

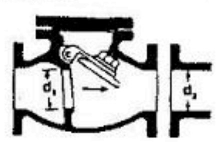
Minimum pipe velocity for full disc lift
$= 55\, \beta^2 \sqrt{\bar{V}}$

If:
$\beta = 1 \ldots K_1 = 200\, f_T$
$\beta < 1 \ldots K_2 =$ Formula 7

Minimum pipe velocity for full disc lift
$= 75\, \beta^2 \sqrt{\bar{V}}$

SWING CHECK VALVES





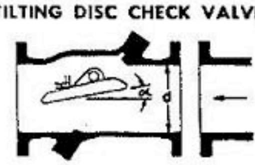
$K = 100\, f_T$

Minimum pipe velocity (fps) for full disc lift
$= 35 \sqrt{\bar{V}}$

$K = 50\, f_T$

Minimum pipe velocity (fps) for full disc lift
$= 60 \sqrt{\bar{V}}$ except
U/L listed $= 100 \sqrt{\bar{V}}$

TILTING DISC CHECK VALVES



	$\alpha = 5°$	$\alpha = 15°$
Sizes 2 to 8″ ... $K =$	$40\, f_T$	$120\, f_T$
Sizes 10 to 14″ ... $K =$	$30\, f_T$	$90\, f_T$
Sizes 16 to 48″ ... $K =$	$20\, f_T$	$60\, f_T$
Minimum pipe velocity (fps) for full disc lift $=$	$80 \sqrt{\bar{V}}$	$30 \sqrt{\bar{V}}$

STANDARD ELBOWS

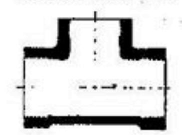
90°

$K = 30\, f_T$

45°

$K = 16\, f_T$

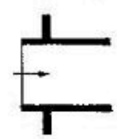
STANDARD TEES



Flow thru run .... $K = 20\, f_T$
Flow thru branch .... $K = 60\, f_T$

PIPE ENTRANCE

Inward Projecting

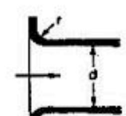


$K = 0.78$

r/d	K
0.00*	0.5
0.02	0.28
0.04	0.24
0.06	0.15
0.10	0.09
0.15 & up	0.04

*Sharp-edged

Flush



For K, see table

PIPE EXIT

Projecting

$K = 1.0$

Sharp-Edged

$K = 1.0$

Rounded

$K = 1.0$

Calculation of the total resistance coefficient for the relief line from the target vessel to vent isolation box: reference diameter 1.5 inch

Component	Resistance Coefficient K
8 feet pipe	1.34
3 - 90° elbows	1.89
2 - 45° elbows	0.68
1 – standard tee (flow through branch)	1.26
1 – relief valve*	0.82
TOTAL	5.99

* according to manufacturer

Calculation of the total resistance coefficient for the relief line from the vacuum vessel to vent isolation box: reference diameter 4.0 inch

Component	Resistance Coefficient K
8 feet pipe	0.41
3 - 90° elbows	1.53
1 – standard tee (flow through branch)	1.02
1 – standard tee (flow through run)	0.34
1 – rupture disk	1.00
1 – pipe exit	1.00
TOTAL	5.30

Calculation of the total resistance coefficient for the relief vent line in shed: reference diameter 4.0 inch

Component	Resistance Coefficient K
30 feet pipe	1.53
4 – 90° elbows	2.04
1 – swing check valve	1.70
1 – pipe entrance	0.50
1 – pipe exit	1.00
TOTAL	6.77

Conversion to 1.5 inch reference diameter pipe:

$$K_a = K_b \left(\frac{d_a}{d_b} \right)^4 = 6.77 \left(\frac{1.5}{4.0} \right)^4 = 0.13$$

Calculation of the total resistance coefficient for the relief vent line in ER2: reference diameter 6.0 inch

Component	Resistance Coefficient K
100 feet pipe	3.00
4 – 90° elbows	1.80
1 – swing check valve	1.50
1 – pipe entrance	0.50
1 – pipe exit	1.00
TOTAL	7.80

Conversion to 1.5 inch reference diameter pipe:

$$K_a = K_b\left(\frac{d_a}{d_b}\right)^4 = 7.80\left(\frac{1.5}{6.0}\right)^4 = 0.03$$

Conversion to 4.0 inch reference diameter pipe:

$$K_a = K_b\left(\frac{d_a}{d_b}\right)^4 = 7.80\left(\frac{4.0}{6.0}\right)^4 = 1.54$$

Total Resistance Coefficients:

A) Target vessel: reference diameter 1.5 inch

From target vessel to vent isolation box:		K = 5.99
From vent isolation box to outside:		
	In shed	K = 0.13
	In ER2	K = 0.03
TOTAL	In shed	K = 6.12
	In ER2	K = 6.02

B) Vacuum vessel: reference diameter 2.5 inch

From vacuum vessel to vent isolation box:		K = 5.30
From vent isolation box to outside:		
	In shed	K = 6.77
	In ER2	K = 1.54
TOTAL	In shed	K = 12.1
	In ER2	K = 6.84

Maximum Pressure Rise as a Function of Resistance Coefficient *K* in Target and Vacuum Vessels:

	Target Vessel	Vacuum Vessel
	Pipe ID 1.5 inch w = 0.2 lb/sec	Pipe ID 4.0 inch w = 0.5 lb/sec
	p_{max} (psia)	p_{max} (psia)
K = 6	38.6	
K = 8	43.0	19.8
K = 10	46.8	20.8
K = 15	55.4	23.3
K = 20		25.5

Conclusion

- The 1.5 inch ID relief line from the target vessel is able to handle a mass flow rate w = 0.2 lb/s with a pressure build-up of no more than 43 psia.

- The 4.0 inch ID relief line from the vacuum vessel is able to handle a mass flow rate w = 0.5 lb/s with a pressure build-up of no more than 23 psia.